Cross-border capacity planning in air traffic management under uncertainty

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Abstract

In European air traffic management (ATM), it is an important decision how much capacity to provide for each airspace, and it has to be made weeks or even months in advance of the day of operation. Given the uncertainty in demand that may materialize until then along with variability in capacity provision (e.g., due to weather), Airspace Users could face high costs of displacements (i.e., delays and re-routings) if capacity is not provided where and when needed. We propose a new capacity sharing scheme in which some proportion of overall capacities can be flexibly deployed in any of the airspaces of the same alliance (at an increased unit cost). This allows us to hedge against the risk of capacity underprovision. Given this scheme, we seek to determine

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the optimum budget for capacities provided both locally and in cross-border sharing that results in the lowest expected network costs (i.e., capacity and displacement costs).

To determine optimum capacity levels, we need to solve a two-stage newsvendor problem: We first decide on capacities to be provided for each airspace, and after uncertain demand and capacity provision disruptions have materialized, we need to decide on the routings of flights (including delays) as well as the sector opening scheme of each airspace to minimize costs. We propose a simulation optimization approach for searching the most cost-efficient capacity levels (in the first stage), and a heuristic to solve the routing and sector opening problem (in the second stage), which is \( \mathcal{NP} \)-hard. We test our approach in a large-sized simulation study based on real data covering around 3,000 flights across Western European airspace. We find that our stochastic approach significantly reduces network costs against a deterministic benchmark while using less computational resources. Experiments on different setups for capacity sharing show that total costs can be reduced by over 8% if capacity is shared across borders - even though we require that no airspace can operate lower capacities under capacity sharing than without (this is to avoid substitution of expensive air traffic controllers with those in countries with a lower wage level). We also find that the use of a common technology provider is a major obstacle to reap the benefits from capacity sharing, and that sharing capacities across airspaces of the same country may instead be preferred.

**Keywords:** air traffic management; capacity planning; simulation optimization

## 1 Introduction

We study a two-stage newsvendor problem in the context of air traffic management with cross-border capacity sharing. In the first stage, a network manager needs to decide on capacity levels to be delivered by each capacity provider for their respective airspaces locally, as well as on a (more expensive) cross-border capacity that can be flexibly deployed in multiple airspaces. Cross-border capacity sharing is intended to address underprovisions caused by variations in demand (e.g., through major traffic shifts in the network) and capacity provision (e.g., through weather events or unplanned air traffic controller shortages). In the second stage, flight intentions by aircraft operators materialize and various sources of uncertainty are being resolved. The network manager needs to decide on demand management measures (delay or re-routing) and on the exact sector opening scheme for each airspace to minimize overall costs.
This problem is motivated by various on-going endeavors that advocate a stronger role of the network manager (NM) as well as capacity sharing between air navigation service providers (ANSPs). At present, European ANSPs rely on their individual resources to deliver required capacity levels to meet demand on a day of operation. These resources mostly refer to the number of available air traffic controllers (ATCOs), and since their rosters and shifts are planned several weeks in advance, there are usually only limited options to call in more ATCOs on short notice when additional capacity is needed. In fact, air traffic control capacity and staffing was cited as one of the main reasons for delays in European airspace in 2018 and 2019 (Eurocontrol (2019, 2020a)). More specifically, about 25% of the 17 million minutes of en-route air traffic flow management (ATFM) delays—i.e., delays imposed by the NM—were attributed to staffing in 2019, with an indication that the lack of ATCOs could have an even higher impact than reported (Eurocontrol, 2020b).

One of the proposed remedies to recurring staffing issues is resource sharing between ANSPs (related to the “capacity-on-demand” concept), which aims to increase resilience to disruptions by enabling a more dynamic temporary delegation of the provision of air traffic services to an alternate provider with spare capacity. The concept forms an integral part of the proposal for the future architecture of the European airspace (SESAR Joint Undertaking, 2019). The Wise Persons Group (2019) also recommends the development of “capacity-on-demand” services, emphasizing the need for improved flexibility and resilience in capacity provision. In fact, cross-border capacity sharing is already at the early implementation stage in Europe, with ANSPs from Finland and Estonia forming an alliance called FINEST. Furthermore, a SESAR project led by ENAIRE (the Spanish ANSP) has tested the sharing of resources among the Madrid and Sevilla area control centers (ACCs), and between the Palma and Barcelona ACCs; they find that the delegation of ATM services is operationally feasible for low and medium traffic periods (SESAR Joint Undertaking, 2022).

More recently, three European ATM doyens welcome the regulation proposal which strengthens the role of the NM in European ATM by means of, inter alia, “putting the NM in a position to manage the capacity brokering process, including the possibility to facilitate delegation of airspace” (Andribet et al., 2022). Moreover, they suggest that the NM role should be further empowered with four key roles, including the role of capacity manager, which would, “based on pan-European demand-capacity balance analysis, decide on the best measures for a better balance including mandatory delegation of airspace from congested ANSPs to less congested neighboring ANSPs” (Andribet et al., 2022).

Therefore, we focus on a strengthened role of the NM to act as a central decision maker...
and investigate different settings of capacity sharing in order to gain insights into what might be promising avenues for future European ATM. It is important to emphasize that capacity sharing is not intended to reduce the cost of capacity provision (which differs dramatically between countries in Europe); instead, it is intended to reduce the cost of re-routings and delays stemming from underprovision due to unforeseen events. Clearly, a proposal of substituting ATCOs in a high-wage country with controllers from a low-wage country will likely not be politically acceptable.

Our main contributions are three-fold: (1) we develop a stochastic solution approach to the hard newsvendor problem underpinning capacity planning based on a simulation optimization framework from Andradóttir and Prudius (2010) and a routing heuristic by Künnen and Strauss (2022); (2) we formulate the problem setting with capacity sharing and adjust the solution approach accordingly; and (3) we inform decision makers on how to implement capacity sharing based on results from a large case study using real flight and network data. We find that our stochastic approach significantly improves, as one would expect, on a benchmark that optimizes capacities based on deterministic problem instances. In contrast to existing approaches, our methodology is also suitable to assess the benefits of capacity sharing: since we use capacity sharing to hedge against the risk of capacity underprovision (depending on materialized traffic and weather), its value can only be assessed stochastically across scenarios, and not deterministically for each scenario individually. Despite the strong self-imposed constraint that every airspace has to have at least as much capacity under capacity sharing than without (to make the concept politically more acceptable), we still find that overall savings of over 8% are realistic.

The paper is organized as follows: We review the relevant literature in §2 and provide a formal description of the problem in §3. In §4, we present a stochastic optimization approach to solve the problem efficiently and discuss a method to evaluate capacity sharing. In §5, we compare the stochastic approach against a deterministic benchmark on a realistically-sized case study. We also provide different design options for capacity sharing and test them using the proposed methodology. We close with recommendations in §6.

2 Literature Review

Capacity sharing

The idea of capacity resource sharing is not entirely new, but to the best of the authors’ knowledge, there are no academic papers which explore the potential benefits of such a con-
cept in ATM. While Ivanov et al. (2019) evaluate European ATM with a strengthened role for the network manager, they only argue that capacity sharing could improve the cost-efficiency of the system, and do not explore this option further. The report of the European Commission (2020) distinguishes three implementation settings for capacity sharing as part of the future European airspace architecture, namely: non-tactical, time-critical and virtual center settings. Some arrangements in Europe already show certain features of these settings. Examples include the Finnish-Estonian dynamic cross-border sectorisation (FINEST), the planned provision of ATM services by the Maastricht Upper Area Control Center for SloveniaControl, and the operation of a virtual center serving both Zurich and Geneva ACCs (European Commission, 2020). In our work, we investigate capacity sharing designs aligned with both the time-critical and virtual center settings and compare it with the setting without capacity sharing.

While we are not aware of any academic publications on capacity sharing in ATM, there is work in electricity markets that analyzes cross-border capacity balancing, see Baldursson et al. (2018). They consider three settings concerning sharing agreements: either no sharing at all, exchange of electricity across borders and lastly a common reserve that multiple countries can draw on (similar to our considered designs for capacity sharing). They find that a focus on overall social welfare (rather than the individual players’ costs) is important to incentivize collaboration. This aligns with our choice of assuming a central planner seeking to minimize total costs whilst ensuring that no provider operates less capacity than without capacity sharing.

**Newsvendor problem**

Structurally, the planning problem that we consider is known as newsboy or newsvendor problem in the inventory management literature. The basic version of this problem is to decide on what quantity of products to order so as to sell them later at an a priori unknown profit (since this will depend on materialization of uncertain demand). Various variations have been studied over the past decades, see Khouja (1999) and Qin et al. (2011) for reviews. In our context, we assume that demand for air traffic services is independent of our capacity decision such that we do not need to model interactions between the two. This seems justifiable since, in practice, demand is largely driven by flight schedules which in turn depend on end customer demand and airport slot availability (but not on en-route airspace capacity). We also make the simplifying assumption that the unit price of capacity, which we measure in sector-hours, is constant and only differentiated based on whether it can be deployed only
locally or for capacity sharing. Another feature of newsvendor problems is how the risk attitude of the decision maker is modeled. We settle on the classic assumption of risk-neutrality (meaning that we optimize expected costs), because we assume that the capacity decisions are modeled for a short period only (e.g., a single day) and would be taken frequently.

**Sample Average Approximation**

The main challenge in solving our problem is that the evaluation of the expectation is expensive. For such problems, sample average approximation (SAA) had been proposed by Kleywegt et al. (2002), who also show that the optimal solution based on SAA converges to the true optimal value with probability 1. The idea is essentially to replace the expectation with a finite sample average – that then represents a deterministic function – and that accordingly can be minimized using deterministic optimization methods. In particular, SAA approaches usually require estimates of the gradient of the objective function so as to employ gradient-based numerical optimization techniques. For an introduction to SAA approaches, see Kim et al. (2015). In our application, the objective is discontinuous and sub-gradients would be computationally very expensive so that we need a derivative-free approach. Theoretical properties (in particular, bounds on SAA’s accuracy) of a data-driven newsvendor were more recently studied by Levi et al. (2015). We adopt a random search method called the ‘asymptotically optimal set (AOS) framework’ of Hu and Andradóttir (2019) that has been designed specifically for minimization of an expectation over a discrete or continuous domain that cannot be evaluated exactly. This approach does not require gradients and has the attractive theoretical feature that, as the name suggests, it ensures asymptotic optimality in that the best point in the candidate set converges almost surely to the global optimum, and other sub-optimal candidates will ultimately be eliminated from the candidate set with probability 1. It improves on the ‘adaptive search with resampling’ approach of Andradóttir and Prudius (2010) by including a method of discarding inferior points from the pool of candidates; this is particularly important in our application since the computational effort of re-evaluating a solution is significant.

**Deterministic approach**

In a model closest to our work, Starita et al. (2020) tackle the same strategic capacity planning model for European ATM. They propose a heuristic approach to determine capacity levels for a given demand and capacity scenario. The authors also develop different policies that define how the optimal capacity levels for each scenario (also referred to as “capacity budgets”) can
be combined into one capacity decision. In contrast to their approach - which starts with a number of known scenarios of the future and results in a suggested capacity budget - we start from the capacity budget and work towards better budgets by assessing the quality of each budget on random scenarios. In this process, we focus most computational effort on the most promising solutions, while allowing new budget decisions to be discovered and investigated. Approaching the problem as a stochastic problem is particularly important since we want to study capacity sharing across airspaces as a means to hedge against the risk of capacity underprovision. Using the approach of Starita et al. (2020) would mean that we start with a deterministic flight scenario with all uncertainties resolved; in that case, there is no need anymore for (the more expensive) cross-border control since we can simply adjust the local capacities accordingly. As we demonstrate in the numerical results in §5, this deterministic approach leads to much worse decisions. To the best of our knowledge, no other research has yet been carried out to study the strategic capacity planning problem.

3 Problem Statement

In this section, we define the two-stage newsvendor problem under consideration. We first discuss the domain of the problem in the first stage, both with and without capacity sharing. We then show that the evaluation of the objective at the second stage is both noisy and expensive and thus set the stage for the proposed simulation optimization in the next section.

3.1 First stage: Search space and conditions for capacity sharing

We consider the problem \( \min_{x \in X} f(x) \), where \( f(x) = E_S(G(x, S)) + c^T x \); \( x \) is a vector of capacity allocations to airspaces (measured in sector-hours) out of the finite set of designs \( X \); \( S \) represents a random scenario for the materialization of demand and potential capacity provision disturbances; \( G(x, S) \) is the displacement cost function given scenario \( S \) and capacity budgets \( x \); and the capacity unit costs are given in vector \( c \). Displacement costs represent the cost of re-routing and/or delaying flights. The exact evaluation of \( f(x) \) is impossible due to the expectation over a complex distribution of scenarios, combined with the fact that even a single objective function observation \( G(x, S) + c^T x \) is expensive, since \( G(x, S) \) is a large integer program. Moreover, \( G(x, S) \) is not continuous in \( x \), which means that we are not able to construct derivatives at all feasible solutions.

The domain \( X \) of potential solutions can be considered to be finite because for each unit time interval (e.g., 30 minutes), each airspace has only a finite number of potential
configurations that it can operate, and each configuration corresponds to a fixed number of sector-hours. In other words, one can enumerate all combinations of configurations that an airspace may operate over the course of a day, and correspondingly would know the number of sector-hours required for each such combination, meaning that the optimal solution must be in this finite set. Of course, the cardinality of $\mathcal{X}$ is very high and therefore it is computationally intractable to explore the entire domain.

However, we do not need to explore the entire domain since, at the strategic planning stage, we already have information on flights from scheduled carriers; uncertainty in the spatio-temporal distribution of demand mainly stems from non-scheduled flights and capacity provision disruption (due to adverse weather or ATCO shortages). Non-scheduled flights usually amount to no more than 20% of total traffic, so the majority of traffic is known in advance and the overall pattern in terms of likely congested airspaces is known. This allows us to define a sensible search space $\mathcal{X}$ within certain maximum and minimum bounds of sector-hours along each dimension. In practice, the range of sector-hours to consider for each airspace can be reliably reduced to around 50, giving a practical size of the search space of $50^{|A|}$ (where $|A|$ is the total number of airspaces). Furthermore, with 80% of traffic known in advance, we find that the number of scenarios required to establish sensible estimates for $f(x)$ is around 300, in a practical instance with 3,000 flights.

In the following, we assume that this process has been completed to identify domain $\mathcal{X}$. We then need to identify the best solution $x^* \in \mathcal{X}$ with a limited computational budget that we measure in terms of the maximal number of objective function evaluations.

We need some additional notation in order to fully specify the objective function. First, a solution $x := [(x_a)_{a \in A}; (x^0_a)_{a \in A}]$ consists of sector-hour budgets $x_a$ for each airspace $a$ that can only be used for that airspace, as well as sector-hour budgets $x^0_a$ that can be used in any airspace within the alliance that $a$ is part of. Let alliance $g \in G$ consist of airspaces $a \in A_g \subseteq A$, where $G$ denotes the index set of all such alliances. We allow $A_g = A$ since capacities may theoretically be shared across the entire network. With $x_a$ and $x^0_a$ respectively, we distinguish between the physical allocation of capacity within the headquarters of an ANSP in an airspace (meaning where the controller would actually be based), and where their workforce is being virtually deployed. This difference is important since we assume, perhaps conservatively, that cross-border capacity sharing would only be politically acceptable if no ANSP would have labor displaced, i.e. if each airspace $a$ has at least as many sector-hours physically assigned as they would have if there was no capacity sharing at all. In particular, we require that each airspace’s total capacity budget must be no less than the threshold
capacity \( \bar{x}_a \) (which is calculated by a previous model run without capacity sharing and serves as a static parameter in the model):

\[
x_a + x_a^0 \geq \bar{x}_a \quad \forall a \in A.
\] (1)

Finally, since the introduction of capacity sharing capability will lead to additional training and license requirements of ATCOs, those controllers eligible to work in other airspaces will need to be compensated at a higher rate. Therefore, we require \( c_a^0 > c_a \), where \( c_a^0 \) denotes the cost per sector-hour of providing capacity in any airspace \( a' \neq a \) by an ATCO in airspace \( a \). Since the majority of cost associated with capacity sharing are fixed cost (e.g., from training ATCOs), they occur irrespective of whether the ATCO actually works in a non-local airspace \( a' \); the same applies to \( c^0 \).

### 3.2 Second stage: Evaluation of displacement costs

In order to evaluate any capacity decision \( x \), we need to determine in the second stage the displacement cost \( G(x, S) \) across a range of scenarios \( S \) (which reflect the uncertainty in traffic and capacity provision). Recall that displacement costs refer to the cost of re-routing and/or delaying flights. For a given capacity \( x \) and scenario \( S \), function \( G(x, S) \) represents the minimum displacement costs generated by routing all flights in scenario \( S \) (which we denote by \( F^S \)) across the air traffic network constrained by capacities \( x \). Therefore, in order to determine \( G(x, S) \) we need to jointly decide on the allocation of shared capacity to each airspace, the sector opening scheme and the routing of flights.

To model the allocation of shared capacities to individual airspaces, we denote by \( h^0_a \) the number of sector-hours that will be virtually deployed in airspace \( a \), and require

\[
\sum_{a \in A_g} h^0_a \leq \sum_{a \in A_g} x_a^0 \quad \forall g \in G.
\]

To illustrate this point, \( x_a^0 = 10 \) would mean that airspace \( a \) plans for 10 sector-hours of capacity sharing at their location which can then be flexibly used in any airspace \( a' \in A_g \) within the alliance of which airspace \( a \) is a member.

Before we present the complete mathematical model on sector opening and routing, we introduce the necessary notation. The sector opening scheme specifies which airspace operates under which capacity setup at each time. Let each airspace \( a \) consist of non-overlapping elementary sectors \( e \in E^a \). Depending on traffic, these elementary sectors can be combined.
into larger, collapsed sectors $l$ in pre-defined ways. The way in which sectors can be combined is governed by configurations. Any configuration $c \in C^a$ represents a partitioning of airspace $a$ into collapsed sectors $l \in L^c$; the elementary sectors contained in any sector $l$ are in turn stored in $E^l$. Since each elementary or collapsed sector is individually controlled by ATCOs, more collapsed sectors require more ATCOs and thus more sector-hours. Let $\bar{h}_{ac}$ denote the number of sector-hours required to run configuration $c$ in airspace $a$ for one time unit. Furthermore, let the day of operations be divided into equally-sized, discrete operating time intervals $u \in U$ (we choose an interval of 30 minutes as this is the usual time required to change a configuration). We then have $\bar{h}_{ac} = |L^c|/2$ since $|L^c|$ is the number of sectors under $c$ and each time unit is half an hour long. Finally, we denote by $\kappa^S_l$ the capacity of each sector $l$ under scenario $S$, which corresponds to the maximum flights that can cross the sector in a single time interval. Note that the sector capacities depend on scenario $S$ because uncertainty in capacity provision (e.g., due to weather) can reduce nominal capacity levels.

To model the routing of flights, we denote by $R^f$ the set of all possible routes for flight $f$, which always includes the shortest trajectory of the flight alongside various delay and rerouting options. We define each route $r \in R^f$ as a sequence of elementary sector- and time-combinations. For this purpose, we denote by $(b)_{f,r,e,u}$ the sector-route incidence matrix, where $b_{f,r,e,u}$ is 1 if flight $f$ on route $r$ uses elementary sector $e$ at time $u$, and 0 otherwise. For each route $r \in R^f$, let $d_r^f$ represent the displacement costs generated by routing flight $f$ through $r$, which are determined net of the shortest trajectory at no delay.
We can then define the displacement cost function $G(x, S)$ as follows:

$$G(x, S) = \min_{y, z} \sum_{f \in F^S} \sum_{r \in R^f} d_f^r y^f_r$$

subject to:

$$\sum_{a \in A^g} h^0_a \leq \sum_{a \in A^g} x^0_a \quad \text{for } g \in G \quad \text{(2)}$$

$$\sum_{a \in A} \sum_{c \in C^a} \bar{h}_{ac} z_{acu} \leq x_a + h^0_a \quad \text{for } a \in A \quad \text{(3)}$$

$$\sum_{f \in F^S} \sum_{r \in R^f} \sum_{e \in E^l} b_{freu} y^f_{ru} z_{acu} \leq \kappa^l_S \quad \text{for } a \in A, c \in C^a, l \in L^c, u \in U \quad \text{(4)}$$

$$\sum_{c \in C^a} z_{acu} = 1 \quad \text{for } a \in A, u \in U \quad \text{(5)}$$

$$\sum_{r \in R^f} y^f_r = 1 \quad \text{for } f \in F \quad \text{(6)}$$

$$h^0_a \in \mathbb{N}^+ \quad \text{for } a \in A$$

$$y^f_r \in \{0, 1\} \quad \text{for } f \in F, r \in R^f$$

$$z_{acu} \in \{0, 1\} \quad \text{for } a \in A, c \in C^a, u \in U.$$
4 Simulation Optimization Approach

Our goal is to find a capacity budget $x^*$ that we confidently believe to lead to low expected network costs across random scenarios $S \in \mathcal{S}$. In the following, we denote by $f(x, S)$ the network costs generated by budget $x$ in scenario $S$, i.e., $f(x, S) = G(x, S) + c^T x$. Here, any scenario $S = (F^S, W^S)$ constitutes a materialization of traffic $F^S$ due to unknown demand from non-scheduled flights, and capacity $W^S$ due to uncertain provision of capacity services. Note that $W^S$ impacts both the actual available sector-hours $x^S$ (due to ATCO shortages) and actual sector capacities $\kappa^S$ (due to weather). To find a high-quality capacity decision $x^*$ as outlined above, we use a framework that balances the exploration of new budgets with the exploitation of existing budgets (meaning to measure $f(x, S)$ for promising candidates $x$ on additional scenarios $S$ to increase our confidence in the average performance); see §4.1. We propose an efficient method to evaluate $f(x, S)$ in each measurement step, see §4.2.

4.1 Framework

We base our approach on the Asymptotically Optimal Set framework by Hu and Andradóttir (2019), which we adjust to our problem. The procedure seeks to determine an optimal capacity decision $x^*$ given solution space $\mathcal{X}$, and is summarized in Algorithm 1. The idea is that we iteratively develop a pool $\mathcal{X}_j^* \subset \mathcal{X}$ of promising solutions (where $j$ is the number of sampled solutions $x$), which is reduced to decision $x^*$ over time. In each iteration $i$ we either evaluate a new candidate $x \in \mathcal{X} \setminus \mathcal{X}_j^*$, or re-assess an existing candidate from this pool on a new scenario $S \in \mathcal{S}$.

To trade off exploration with exploitation, we need to decide how frequently we sample new solutions. Over time we want to explore new solutions less frequently to ensure that more effort can be spent on evaluating the quality of encountered promising solutions. As suggested by Hu and Andradóttir (2019), we sample new solutions at iterations $i = [j^{1.5}]$, and re-sample existing solutions otherwise.

At iterations $i$ where $i \neq [j^{1.5}]$, we sample a new candidate $x$ according to a pre-defined sampling strategy. To ensure that we cover the entire solution space, we use Latin Hypercube Sampling, which is described in the Appendix. We then need to decide whether to accept $x$ into the pool of promising candidates, or whether to discard it. For this purpose, we define a benchmark scenario $S_1$ without disturbances, i.e., we set $|F^{S_1}|$ to the average number of expected flights, and $W^{S_1}$ such that $x^{S_1} = x$ and $\kappa^{S_1} = \kappa$. We then accept a solution $x$ into the pool if the objective function value $f(x, S_1)$ is at most $\lambda$ worse than our current best candidate, i.e., $f(x, S_1) - \hat{f}_i(x^*_i) < \lambda$, where $\hat{f}_i(x)$ denotes the average objective value of $x$.
**Algorithm 1** Exploration-exploitation framework to seek optimum capacity

Input: Scenarios $\mathcal{S}$, solution space $\mathcal{X}$, sampling and re-sampling strategy

1: Initialize: $i = 0$, $j = 1$, $\mathcal{X}^*_0 = \emptyset$, $\hat{f}_0(x^*) = M$ (large number)
2: while $\exists x \in \mathcal{X}^*_j : N(x) < N^{\text{max}}$ do
3: \hspace{1em} $i = i + 1$
4: \hspace{1em} if $i = \lfloor j^{1.5}\rfloor$ then
5: \hspace{2em} Select $x \in \mathcal{X}$ based on sampling strategy, evaluate $f(x, S_1)$ and set $j = j + 1$
6: \hspace{2em} if $f(x, S_1) - \hat{f}_i(x^*) < \lambda$ then
7: \hspace{3em} $\mathcal{X}^*_j = \mathcal{X}^*_{j-1} \cup \{x\}$, $\hat{f}_i(x) = f(x, S_1), N(x) = 1$
8: \hspace{2em} else
9: \hspace{3em} $\mathcal{X}^*_j = \mathcal{X}^*_{j-1}$
10: \hspace{1em} end if
11: \hspace{1em} for $x \in \mathcal{X}^*_j$ (ensure minimum amount of re-sampling) do
12: \hspace{2em} if $N(x) < \lceil j^{0.5}\rceil$ then
13: \hspace{3em} Set $n = \lfloor j^{0.5}\rfloor - N(x)$ and evaluate $f(x, S_{N(x)+1}), \ldots, f(x, S_{N(x)+n})$
14: \hspace{3em} Set $\hat{f}_i(x) = \frac{\hat{f}_{i-1}(x)N(x) + \sum_{k=1}^{n} f(x, S_{N(x)+k})}{N(x)+n}$, and $N(x) = N(x) + 1$
15: \hspace{1em} end if
16: \hspace{1em} end for
17: \hspace{1em} for $x \in \mathcal{X}^*_j$ (discard poor solutions) do
18: \hspace{2em} if $\hat{f}_i(x) - \hat{f}_i(x^*) > \lambda/j^{0.15}$ then
19: \hspace{3em} $\mathcal{X}^*_j = \mathcal{X}^*_j \setminus \{x\}$
20: \hspace{2em} end if
21: \hspace{1em} end for
22: else
23: \hspace{1em} Select $x \in \mathcal{X}^*_j$ based on re-sampling strategy and evaluate $f(x, S_{N(x)+1})$
24: \hspace{1em} Set $\hat{f}_i(x) = \frac{\hat{f}_{i-1}(x)N(x) + f(x, S_{N(x)+1})}{N(x)+1}$, and $N(x) = N(x) + 1$
25: \hspace{1em} Update $x^*_i = \arg\min_{x \in \mathcal{X}^*_j} \hat{f}_i(x)$ and
26: \hspace{1em} end if
27: \hspace{1em} end while

Output: Capacity budget $x^*_i$ with network cost $\hat{f}_i(x^*)$

At iteration $i$. Furthermore, if a sufficient amount of objective function evaluations have been conducted, we discard existing solutions based on a rejection threshold. If $j$ candidates have been evaluated, we discard solutions $x \in \mathcal{X}^*_j$ if $\hat{f}_i(x) - \hat{f}_i(x^*_{i-1}) > \lambda/j^{0.15}$. This threshold is used to remove candidates from the pool which no longer look promising. Note that $\lambda/j^{0.15}$ gets increasingly stringent so as to reduce the pool eventually to the optimal solution. To ensure convergence, a minimum amount of re-sampling is required throughout the procedure. We require for the number of evaluations $N(x)$ of any candidate $x$ that $N(x) \geq \lceil j^{0.5}\rceil$, and determine further objective function values for $x$ if the condition is not fulfilled.
At iterations $i$ where $i \neq \lfloor j^{1.5} \rfloor$, we re-sample an existing candidate $x \in \mathcal{X}_j^*$ according to a pre-defined re-sampling strategy. We use an epsilon greedy approach: we sample with probability $\sigma(j) := 0.99/j^{0.5}$ a solution $x \in \mathcal{X}_j^*$ randomly, and with probability $1 - \sigma(j)$ we choose $x := x_j^*$ (where $x_j^*$ is the best currently known average cost solution). We then evaluate $f(x, S)$ for a randomly chosen scenario $S$ and update average objective function value $\hat{f}(x)$ and evaluation count $N(x)$ accordingly. We stop the procedure if we have evaluated all candidates in the current pool over $N_{\text{max}}$ scenarios. We choose this stopping condition because a) we do not observe large variations in $\hat{f}(x)$ after a certain amount of evaluations, and b) there are often many very good candidates with similar $\hat{f}(x)$ in the pool (for large $j$), so reducing the pool further until only one candidate remains is not beneficial.

One advantage of the proposed framework is that only two parameters ($\lambda$ and $N_{\text{max}}$) need to be set according to the problem at hand (see §5.1). However, the effectiveness and efficiency of the procedure depend on how well and how fast we can evaluate network cost $f(x, S) = G(x, S) + c^T x$ for any budget $x$ and scenario $S$, which we discuss below.

4.2 Cost Evaluation for Given Solution and Given Scenario

To evaluate $G(x, S)$ for any candidate capacity budget $x$ under traffic and capacity scenario $S$, we need to solve the integer program that underpins $G(x, S)$. However, this problem is $\mathcal{NP}$-hard since it represents an instance of the Multidimensional Multiple Choice Knapsack Problem (MMKP), as shown by Künnen and Strauss (2022). In fact, in order to determine an exact solution, we would need to solve an MMKP (i.e., the routing problem) for each of the $\prod_a |C_a|^{\vert U \vert}$ possible combinations of airspace configurations. This is infeasible for realistically-sized instances, and takes already 50 minutes for a small instance with 200 flights across five airspaces in one hour (see Künnen and Strauss (2022)). A potential approach to approximate $G(x, S)$ would be to solve its linear relaxation. However, we do not pursue this option since Starita et al. (2020) demonstrate on a large case study that it delivers poor results for this problem. Instead, we follow the approach in Künnen and Strauss (2022) and separate the sector-opening problem from the routing problem to estimate $G(x, S) \approx D(x, F^S, W^S)$ in polynomial time. More specifically, we conduct two steps to determine $D(x, F^S, W^S)$:

1. Determine best candidate configuration $C'(x, F^S, W^S)$ given the capacity budget $x$, traffic scenario $F^S$ and uncertainty around capacity provision $W^S$, see §4.2.1;

2. Determine the routing of flights with lowest cost $D(C', F^S, W^S)$ given the candidate configuration $C'$, see §4.2.2.
4.2.1 Finding the best candidate configuration

In a first step, we want to determine candidate configuration \( C' = \{ c'_{au} : a \in A, u \in U \} \), which consists of individual configurations \( c'_{au} \) for each airspace and operating time. For this purpose, we first assign each flight to its shortest route, i.e., the route with zero displacement cost; a positive cost is incurred if a flight is “displaced” in time (delayed) or in space (rerouted). We assume that this traffic assignment (represented by allocation \( y \)) gives us a good indication of where capacity is required in the network. Given traffic assignment \( y \), we can compute the capacity shortage \( k_{acu} \) for each airspace \( a \), configuration \( c \in C^a \) and time unit \( u \). We define capacity shortage as the number of flights that exceed sector capacity limits, i.e.,

\[
k_{acu} := \sum_{l \in L} \left( \sum_{e \in E^l} \sum_{f \in F^l} \sum_{r \in R^f} \left( \sum_{b \in B^r} b_{freu} y_{fr}^l - \kappa^S_t \right) \right)^+, \quad \text{where } x^+ := \max\{x, 0\}.
\]

We then determine configuration \( c'_{au} \) for each airspace \( a \) and time \( u \) as the feasible configuration with the lowest total capacity shortage by solving the following ‘configuration integer linear program’ (CILP):

\[
\begin{align}
\text{(CILP)} \quad & \min_z \sum_{a,c,u} k_{acu} z_{acu} \\
\text{s.t.} \quad & \sum_{u \in U} \sum_{c \in C^a} \bar{h}_{acu} z_{acu} \leq x^S_a \quad a \in A \tag{8} \\
& \sum_{c \in C^a} z_{acu} = 1 \quad a \in A, u \in U \tag{9} \\
& z_{acu} \in \{0, 1\} \quad a \in A, c \in C^a, u \in U. \tag{10}
\end{align}
\]

In particular, \( c'_{au} \) is given by the configuration for which \( y_{acu} = 1 \) for each airspace \( a \) and operating time \( u \). To ensure that the problem is always feasible with regards to constraint (8), we require that \( x^S_a \geq |U| \) for all scenarios \( S \) and airspaces \( a \) because at least one sector needs to be operated at each time period. In the numerical experiments, we implement an even stricter lower bound for the airspace capacities, see §5.1.

It is easy to see that the CILP decomposes by airspace; we denote this decomposition by CILP-d. Even after decomposition, the resulting problem still represents a multiple choice knapsack problem (MCKP), which is \( \mathcal{NP} \)-hard. However, as there are typically no more than tens of configuration options per ACC in Europe in each time unit, the problem can be solved exactly in reasonable time. If larger airspaces need to be accounted for, or if the number of operating time units make the problem intractable, we could revert to heuristic approaches for the MCKP, such as the one proposed in Pisinger (1995).

To model capacity sharing, the approach outlined above for determining \( C' \) needs to
be adjusted to allow for sharing of capacities among two or more airspaces. Recall that \( a \in A_g \subseteq A \) \((g \in G)\) represent the airspaces in alliance \( g \) among which capacities can be shared. If capacity sharing is not permitted in certain airspaces, we combine these airspaces in alliance \( g' \) and set \( x^0_{g'} := 0 \). We can obtain the sector opening scheme under capacity sharing by solving the following mixed integer linear program (which we could decompose by sharing alliance \( g \)):

\[
\text{(XCILP)} \quad \min_{h_0, z} \sum_{a,c,u} k_{acu} z_{acu} \tag{11}
\]

\[
\text{s.t. (9), (10)}
\]

\[
\sum_{u \in U} \sum_{c \in C^a} \bar{h}_{ac} z_{acu} \leq x^S_a + h^0_a \quad a \in A \tag{12}
\]

\[
\sum_{a \in A_g} h^0_a \leq \sum_{a \in A_g} x^0_a \quad g \in G \tag{13}
\]

\[
h^0_a \in \mathbb{N}^+ \quad a \in A. \tag{14}
\]

### 4.2.2 Determine lowest displacement costs with given configuration

After having determined configuration set \( C' \), we apply the MMKP-based heuristic proposed in Künnen and Strauss (2022) to determine the routing of flights with lowest displacement costs \( D(C', F^S, W^S) \). The approach is summarized in Algorithm 4 in the Appendix. We initialize the routing procedure by assigning each flight \( f \in F^S \) to the route with lowest displacement costs. Let \( L' = \{l \in L' : c' \in C'\} \) be the sectors defined by \( C' \). To establish a feasible solution, we then iteratively reassign flights on the most congested sector \( l^* \) until all sectors \( l \in L' \) are within capacity limits \( \kappa^S_l \) (which depend on \( W^S \)). To decide which flight to reassign to another route, we compute a decision parameter \( \gamma^f_{cr} \) that weighs the change in displacement costs with the change in capacity usage on the most congested sector. Finally, we test if we can use potential spare capacities to further improve this feasible solution. For that purpose, any flight \( f \) and route \( r \) is reassigned from current route \( r' \) to \( r \), if this reassignment improves displacement costs while keeping the routing feasible.

As shown in Moser et al. (1997), Algorithm 4 has complexity \( O(m(n-o)^2 + mn) \), where \( m = |L'| \) is total number of sectors given configuration \( C' \), \( n = \sum_{f \in F^S} |R^f| \) is total number of flight-route combinations, and \( o = |F^S| \) is total number of flights. Given this complexity, evaluating \( D(C', F^S, W^S) \) is still computationally very expensive for larger networks and traffic scenarios. Therefore, we use Algorithm 4 to generate a large number of observations \( \hat{D} \) given certain capacity budgets, traffic scenarios and capacity uncertainties, and apply a
linear regression to approximate $D$. In particular, we estimate $D$ based on the capacity shortages $k_{ac'u}$ that a certain capacity budget and traffic and capacity scenario generates:

$$
\hat{D}(C', F^S, W^S) = \beta_0 + \sum_{a \in A} \beta_a \sum_{u} k_{ac'u}.
$$

(15)

The idea is that the number of capacity shortages directly influences the required amount of re-allocations of flights to alternative routes, and thus the size of total displacement costs. This choice of explanatory variable also offers another benefit: Since the CILP is decomposable by airspace, the solution with minimum capacity shortage $k_{acu}$ will also be the one with minimum cost estimate (based on (15)). However, the XCILP can only be decomposed by alliance $g$ (not by airspace $a$), because we allow capacities to be shared among airspaces in an alliance. Therefore, we may encounter cases where the solution with minimum capacity shortage does not provide minimum costs. To avoid this, we use parameters $(\beta_a)_{a \in A}$ as weights in the objective function of the XCILP to directly optimize over costs. In particular, we replace function (11) in the XCILP to get:

$$(\text{XCILP}^*) \min_{h, z} \sum_{a, c, u} \beta_a k_{acu} z_{acu}
$$

s.t. (9), (10), (12), (13), (14).

We denote the decomposition by alliances of the XCILP* by XCILP-d*. Overall, the CILP-d (or XCILP-d* for capacity sharing) together with regression (15) provide us with cost estimate $\hat{D}$ for any given capacity budget and scenario. We use this estimate in each iteration of Algorithm 1 to evaluate $f(x, S) \approx \hat{D}(C', F^S, W^S) + c^T x$. Since we can separately determine estimates $\hat{D}$ for each airspace (or alliance in the case of capacity sharing), we also run Algorithm 1 individually for each airspace (or alliance), which significantly speeds up the solving process. To further improve estimates $\hat{D}$, we could re-run Algorithm 4 and update parameters $\beta$ after a fixed amount of iterations, using only budgets $x$ that are close to the current optimum. However, we decide not to apply this updating procedure because (a) re-applying Algorithm 4 consumes sizable computational resources, and (b) the variation in displacement costs across scenarios is too large to warrant a more exact cost estimate per scenario.
5 Numerical Results

The objective of our research is two-fold: (1) We want to assess the performance of our stochastic optimization approach in finding optimal strategic capacity levels, and (2) to guide decision makers towards an operable and effective design of capacity sharing. In order to assess the value of capacity sharing, we need a case study of sufficient size (in terms of time horizon and number of airspaces) such that capacity sharing services may actually be demanded. For this purpose, we use a case study based on real flight data covering a 6-hour time period on the busiest day of 2016, see §5.1. We discuss results of our experiments on our stochastic approach and on capacity sharing in §5.2 and §5.3, respectively.

5.1 Case study description

We use the network data based on the large case study in Starita et al. (2020) and increase the considered time horizon and the number of flights. The real data set covers large parts of en-route airspace in Western Europe and was obtained using Eurocontrol’s Demand Data Repository (DDR2) service. On the capacity side, the case study includes 15 ACCs (i.e., airspaces) across 8 ANSPs, which consist of 177 elementary sectors in total. These elementary sectors are combined in various ways to form a total of 173 different configurations, which were selected among the most frequently used ones in 2016. Figure 1 shows exemplary scheduled traffic across the considered network at a specific point in time. The capacity costs used in the case study are average costs per sector-hour for each ANSP, computed based on Eurocontrol (2018b). We treat these costs as variable costs in the simulation because expanding capacity will make it necessary to hire additional ATCOs in the long run (and vice versa). To define the solution space $X$, we determine the minimum sector-hours $x_a$ and maximum sector-hours $\overline{x}_a$ ($a \in A$) that each airspace operated based on historic data; we then have $X = \{ x \in [x_a, \overline{x}_a]_{a \in A}, x \in \mathbb{N}^{|A|} \}$.

On the demand side, the data includes all flights in the selected network on September 9, 2016, the busiest day in European airspace that year. For the simulation, we restrict the time frame to 9am – 3pm and select the 2,400 scheduled flights crossing the network during this time. Among the remaining flights in the data set, we randomly choose 2,500 to serve as a pool of non-scheduled flights. To create the traffic scenarios, we uniformly sample from this pool the non-scheduled flights, where the number of flights is drawn from a normal distribution with mean of 600 (to generate on average 20% non-scheduled traffic) and standard deviation of 200. Each traffic scenario $F^S$ then consists of all 2,400 scheduled...
flights and a sample of non-scheduled traffic, giving on average 3,000 total flights.

Each flight in the data set has a reference (shortest) route with zero displacement cost and up to 12 different alternative routing options (in the horizontal or vertical plane). Reference and alternative routes were generated using Eurocontrol’s Network Strategic Tool (NEST) based on the last filed flight plan for every flight in a data set. In particular, we identify up to 40 horizontal route options for each flight, and then compute the vertical profile (considering both the actual route network and constraints in the horizontal and vertical planes). Among the resulting route options, the shortest trajectory is then chosen based on lowest total distance; wind conditions are not considered since the capacity decision is made in the strategic phase of ATM in which weather forecasts are still unreliable. We assume that a flight can only be subject to one demand management measure: either delay or re-routing (this also keeps the number of total route alternatives low). Therefore, we add three potential delay options to the shortest route of each flight: 10, 20 and 30 minutes. To further reduce solution time of the model (especially Algorithm 4), we pre-process the routes of all scheduled flights and keep only frequently used route options. Displacement costs are computed for each flight and route option separately and consist of re-routing and delay costs. Re-routing costs include mainly additional fuel costs, crew and passenger costs and are estimated based on Cook and Tanner (2015) and Eurocontrol (2018a). Delay costs depend to a large extent
on the AU’s business model. Thus, let $\omega$ represent a scaling factor for delay cost of each flight based on the AU’s business model $v(f)$: 0.4 for charter flights and low cost carriers, 1 for full-service carriers, 1.75 for business aviation and 10 for flights which are typically exempt from demand management measures. Also, let $d_{fr}^f$ represent delay cost references based on Cook and Tanner (2015) which differ by aircraft type and increase non-linearly with the duration of delay. We can then estimate delay costs $d_{fr}^f$ for all flights $f$ and delay options $r \in R_{delay}$ as follows:

$$d_{fr}^f := \omega v(f) d_{fr}^f \quad r \in R_{delay}^f, f \in F.$$ 

To model a real-life setting, we need to account for both demand- and capacity-side uncertainties. Demand-side uncertainty stems from non-scheduled flights (in terms of total number of such flights and where and when they appear in the network), which is modeled through our traffic scenarios $F^S$ described above. For capacity-side uncertainty $W^S$ we distinguish between internal (i.e., ATCO shortages) and external (i.e., adverse weather) effects. This distinction is particularly important because internal effects can be mitigated, at least partly, through capacity sharing while external effects cannot. To estimate internal effects, we analyze air traffic flow management (ATFM) regulations due to ATCO staffing reasons in the considered network. An ATFM regulation is issued when demand exceeds capacity in an airspace volume for a period of time. We partially rely on analysis of 2016 ATFM regulation data to derive frequencies of staffing regulation occurrences for each airspace in the case study, which we show in the Appendix. External effects can sometimes severely reduce capacities in an affected airspace for a certain amount of time, which is why we apply a more differentiated probability distribution here. Again, based on historical ATFM regulation data, we assume that the capacity of any one elementary sector (and the collapsed sector containing it), is reduced to 90% in 10% of cases, to 70% in another 10% of cases and to 50% in 5% of cases.

Two parameters in Algorithm 1 ($\lambda$ and $N_{max}$) need to be tailored to our case study. Parameter $\lambda$ determines which candidates we accept into the pool and which ones we discard (after a certain amount of evaluations). We decide to only accept candidates that stay within 20% of the current minimal cost solution. Accordingly, we set $\lambda := 2,000$ when running Algorithm 1 for each airspace, and $\lambda := 4,000$ when running it for each alliance (if capacity sharing is allowed). Furthermore, we set $N_{max} := 300$ since we do not observe significant changes in $f(x,S)$ if we evaluate more than 300 scenarios.
5.2 Value of the stochastic approach

5.2.1 Simulation setting and evaluation

To analyze whether the stochastic approach leads to better capacity planning decisions, we compare it with a deterministic benchmark, which we describe below. The quality of a capacity management decision is assessed based on the expected network costs (both displacement and capacity costs) that it creates. We test the approaches on the setting without capacity sharing, i.e., all airspaces can only use their own, local capacities to manage traffic demand. In the stochastic approach we determine optimal capacity budget \( x^* = (x^*_a)_{a \in A} \) using Algorithm 1 as discussed in §4. Here, we use a fixed set of 300 training scenarios (stored in \( S_1 \)) for traffic and capacity uncertainty on which each candidate budget is evaluated.

To establish a benchmark for our stochastic approach, we use the strategic capacity planning model proposed by Starita et al. (2020). In this approach, the optimal budgets \( x^S \) are determined separately for each scenario \( S \in S_1 \) by decomposing the problem underpinning \( G(x, S) \) into a master- and sub-problem. After evaluating all scenarios, the benchmark capacity budget \( x^B \) is determined using the so-called risk-based policy, which performed best in Starita et al. (2020). More specifically, we have \( x^B = (x^S_a)_{a \in A} \) where each \( x^S_a \) is chosen among all \( x^S_a (S \in S_1) \) such that the probability that any other budget \( x^S_a (for S \neq S') \) has higher capacity in airspace \( a \) is less than a given \( \epsilon \) (which is set to 0.05).

Once the capacity budgets \( x^* \) and \( x^B \) have been determined, we assess their performance on a set of 200 testing scenarios (stored in \( S_2 \), and sampled from the same distributions as \( S_1 \)). When applying Algorithm 4 to solve the routing problem, we implicitly assume that the NM can autonomously decide on the routing of each flight (in the strategic phase). However, on the day of operation the decision is made iteratively between the NM and airspace users (AUs). Therefore, we use a discrete-time event simulation to test each capacity decision, in which the NM decides on the sector-opening scheme of each airspace (using the CILP), communicates all feasible trajectories to the AU and the AU then decides on the final trajectory among all feasible options. To model the AU decision, we assume that the AU chooses the trajectory with lowest AU-adjusted displacement cost \( \delta \), which we determine as follows:

\[
\delta_r^f := \begin{cases} 
    d_r^f & \text{if } r \text{ is a re-routing option,} \\
    N(d_r^f, 0.1) & \text{if } r \text{ is a delay option.}
\end{cases}
\]

Here, \( N() \) denotes the draw from the normal distribution. We include this uncertainty for
delayed routes because, in contrast to rerouting costs, delay costs depend more strongly on AU-specific parameters. Note that we do not consider ATM service charges in the trajectory decision because we assume that charges are route-independent (for details on the so-called airport-pair charging principle, see Pavlović and Fichert (2019)).

Furthermore, we introduce capacity uncertainty dynamically in the discrete-time event simulation by updating the available capacity budget \(x^*S\) and sector capacities \(\kappa^S\) after every time period \(u\). In contrast, the materialized demand is fixed for each run of the simulation (and thus not updated dynamically), because we assume that flights are fully known at the beginning of the day of operation. The procedure is summarized in Algorithm 2. In addition to existing notation, we denote by \(F_u\) the flights that depart within \(u\), by \(S_u\) a scenario that models capacity uncertainties within \(u\) and by \(f(r)\) the flight corresponding to route \(r\).

**Algorithm 2** Discrete-time event simulation to test capacity decision.

Input: Flights \(F\), capacity decision \(x^*\), scenarios \(S_2\)

1: Initialize: Iteration counter \(i := 0\), Routing \(R_0^* := \emptyset\)
2: for \(u \in U\) do
3: Sample new scenario \(S_u \in S_2\) and update capacity uncertainty: \(x^*S_u\) (ATCO shortages) and \(\kappa^S_u\) (adverse weather)
4: Run CILP to determine best configuration \(C'(x^*S_u, F, R^*_i, \kappa^S_u)\)
5: for \(f \in F_u\) (sorted chronologically by departure time) do
6: Determine set of feasible routes:
\[
\bar{R}_f := \left\{ r \in R_f : \sum_{e \in E_l} \left( b_{f,r,e,u} + \sum_{j \in R^*_i} b_{f(j),j,e,u} \right) \leq \kappa^S_a \quad \forall a \in A, u \in U, l \in \mathcal{L} \right\}
\]
7: Determine route choice based on adjusted displacement cost \(\delta_f: r^f := \arg \min_{r \in \bar{R}_f} \delta_r^f\)
8: Update sets: \(R^*_{i+1} := R^*_i \cup r^f, i = i + 1\)
9: end for
10: end for

Output: Routing \(R^*_i\) with displacement cost \(D(x^*, F^S, W^S) := \sum_{r \in R^*_i} d^f_{r(r)}\)

### 5.2.2 Discussion of results

Before we discuss the performance of the stochastic and benchmark approaches to minimize network costs, we comment on efficiency and effectiveness of regression (15) to approximate displacement cost \(G(x, S)\) for a given budget \(x\) and scenario \(S\). For this purpose, we compare the cost estimates from regression (15) on 2,280 instances (each instance is a combination of budget and scenario) against costs determined through Algorithm 4 and the CILP. With a mean absolute error of 6,191 (or 6.4\% of average displacement costs) and \(R^2\) of 0.96, we confirm that regression (15) adequately approximates displacement costs for our purpose. To
illustrate the relationship, we show in Figure 2 the correlation between capacity shortages (aggregated across airspaces and time for purpose of illustration) and displacement costs based on Algorithm 4 for all instances. Applying regression (15) instead of Algorithm 4 (together with the CILP-d) to estimate displacement costs significantly reduces our solution time. For the case study at hand, we manage to reduce average run time for one instance from 20 minutes to under one second, including running the CILP-d.

Table 1 summarizes the simulation results for the capacity decisions $x^*$ and $x^{B*}$ of the stochastic and deterministic approach on 200 scenarios, without capacity sharing. The stochastically determined capacities lead to significantly lower network costs than the benchmark capacity decision, since the displacement cost savings of €118,213 (or 53%) outweigh the higher capacity costs. Overall, network costs are reduced by €96,834, or 28%. Figure 3 illustrates the performance of the stochastic vs deterministic capacity decision across 50 of the 200 evaluated scenarios. Not only does the stochastic solution create lower network costs in all 50 scenarios, but it also reduces the variation in cost (which is also reflected in the standard deviations in Table 1). This is important because it implies a more stable network performance and thus higher reliability of service provision to airspace users (e.g., airlines).

The results also show that the number of flights that cannot be assigned to non-dummy routes is 1.4% (on average across scenarios) for the stochastic approach, which shows that almost all flights can be routed within the given time frame and network conditions. For the benchmark, this share increases to 3.8%. Furthermore, under stochastically-determined capacities a lower share of flights need to be rerouted or delayed than under benchmark capacities (see Figure 4). In fact, more than 84% of flights can take the shortest route for
Table 1. Simulation results for solution approaches (n = 200 scenarios).

<table>
<thead>
<tr>
<th>Approach</th>
<th>Cap. cost</th>
<th>Displ. cost</th>
<th>Network cost</th>
<th>Not assigned</th>
<th>Run time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic</td>
<td>140,936</td>
<td>106,112</td>
<td>247,048 ± 39,295</td>
<td>1.4%</td>
<td>126 min.</td>
</tr>
<tr>
<td>Benchmark</td>
<td>119,556</td>
<td>224,325</td>
<td>343,882 ± 67,460</td>
<td>3.8%</td>
<td>3017 min.</td>
</tr>
<tr>
<td>Difference</td>
<td>21,379</td>
<td>-118,213</td>
<td>-96,834 ± 36,558*</td>
<td>-2.4%</td>
<td>-2,891 min.</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>+18%</td>
<td>-53%</td>
<td>-28%</td>
<td>-63%</td>
<td>-96%</td>
</tr>
</tbody>
</table>

* Significant at 95% confidence level.

Figure 3: Network cost across 50 scenarios for stochastic and benchmark approach.

the stochastic approach vs 72% for the benchmark. These figures confirm that stochastic capacity planning can substantially reduce network cost and improve network performance.

Figure 4: Regulations for stochastic vs benchmark approach (n = 200 scenarios).

Next to cost, an important consideration for the network manager (or ANSPs in the case of local decision-making) is the speed with which capacity decisions can be made. With the proposed stochastic approach, we determine the capacity decision \( x^* \) in around two hours, and thus sufficiently fast for practical application. Figure 5 visualizes an exemplary path of
Algorithm 1 to determine the capacity decision $x^*$ (shown for airspace EDUUUTAS). We see that the procedure converges quickly towards a good solution, with only small variations in the recommended capacity level after around 700 iterations. Since we can decompose the problem by airspace, we could also parallelize the process by solving $x_a^*$ for each airspace simultaneously, which would further reduce the run time to around 10 minutes. In contrast, the run time for the benchmark approach exceeds 3,000 minutes (or around 2 days), and the problem cannot be decomposed and parallelized which renders the method less practical in a dynamic environment. It is important to note that the solution time for the stochastic approach does not include the time to estimate parameter $\beta$ for regression (15), which takes around 20 minutes for each of the roughly 2,000 observations of $G(x, S)$. However, this process can be parallelized for every observation, and it would only have to be conducted once for all future capacity decisions.

Figure 5: Exemplary path to determine capacity decision in stochastic approach.

**Sensitivity Analysis**

In order to test how robust the presented results are to changes in traffic intensity, we conduct a sensitivity analysis across three traffic levels: next to medium traffic with on average 3,000 flights used so far, we analyze performance under low traffic with 2,500 flights and high traffic with 3,500 flights. In each case, we continue to sample traffic scenarios $F^S$ from the data set with 80% scheduled and 20% non-scheduled flights. The results in Table 2 show that the stochastic approach outperforms the benchmark by €94,334 (or 36%) under low
traffic and by €67,184 (or 15%) under high traffic. The improvement is lowest under high traffic because airspaces may simply not be able to increase capacities further to manage all traffic. Furthermore, between 3.4–4.5% of flights cannot be assigned to non-dummy routes under benchmark capacities; the values for the stochastic approach are much lower. Even though the run time for the stochastic approach increases to 156 minutes under high traffic, it stays within practical limits and well below the solution time for the benchmark. Finally, next to the benefits in solution quality and time, the proposed stochastic approach features a methodological advantage over existing deterministic approaches: Rather than fixing scenarios and finding appropriate capacities for each, it fixes the capacities first and then determines its performance across a multitude of scenarios. It is this feature that allows us to use the stochastic approach to determine ‘optimal’ capacity levels in the setting with capacity sharing among airspaces, which we discuss in the following.

Table 2. Simulation results for solution approaches across traffic levels (n= 200 scenarios).

<table>
<thead>
<tr>
<th>Model/Traffic flow</th>
<th>Network cost</th>
<th>Not assigned</th>
<th>Run time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Stochastic</strong></td>
<td>166,369</td>
<td>386,800</td>
<td>0.3%</td>
</tr>
<tr>
<td><strong>Benchmark</strong></td>
<td>260,703</td>
<td>453,984</td>
<td>3.4%</td>
</tr>
<tr>
<td>Difference</td>
<td>-94,334 ± 58,048</td>
<td>-67,184 ± 26,841*</td>
<td>-3.1%</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>-36%</td>
<td>-15%</td>
<td>-92%</td>
</tr>
</tbody>
</table>

* Significant at 95% confidence level.

5.3 Value of capacity sharing

5.3.1 Capacity sharing concepts and evaluation

To provide guidance for decision makers on how to design a potential capacity sharing scheme in European ATM, we test three different design options:

1. Capacity sharing across airspaces within the same ANSP (*Cross-ACC*): Every airspace (i.e., area control center, or ACC) within an ANSP can leverage the shared capacity of its ANSP, next to its own capacity, to manage traffic demand.

2. Capacity sharing across ANSPs (*Cross-border*): In this setting, capacity can be shared between any combination of ANSPs, without restrictions due to location or technology.
3. Capacity sharing among ANSPs with the same technology provider (Common tech):

Next to geographic proximity, the use of common technological infrastructure is a major
criterion for sharing of capacities across ANSPs. Therefore, in this setting an alliance
can only be formed among airspaces that use the same air traffic technology provider
(the provider for each airspace is reported in Table 7 in the Appendix).

To model capacity sharing, we apply the (XCILP-d*) in each evaluation of Algorithm 1
to determine the best candidate configuration for each budget \(x\) and alliance \(g\) given solution
space \(X_g\), where \(X_g = \{ x \in [\underline{x}_a, \overline{x}_a]_{a \in A_g}, x^0 \in [\underline{x}_a^0, \overline{x}_a^0], x, x^0 \in \mathbb{N}^{|A_a|}\} \). Parameters \(\underline{x}_a, \overline{x}_a\) for
\(a \in A\) are given by the case study, see §5.1. In order to fulfill condition (1) for budgets \(x\), we
set \(\underline{x}_a^0(x) = (\overline{x}_a - x_a)^+\) and \(\overline{x}_a^0(x) = x_a - x_a\) for \(g \in G\). The definition of \(G\) and \(A_g (g \in G)\) varies
for each of the three design options above. For the cross-ACC setting, we define each ANSP
as a separate alliance among which capacities can be shared. For the cross-border setting,
the definition of alliances is less straight-forward. Since airspaces can be combined freely in
this setting to form an alliance, the number of potential set partitions increases exponentially
with the number of airspaces. Therefore, we restrict the size of a cross-border alliance to
up to two airspaces, and use a structured approach to determine which combination of such
alliances (i.e., set partitions) promises to deliver the best cost performance. Let \(G\) contain
all 1- and 2-tuples of airspaces \(a \in A\), i.e. \(G := \{ a \in A \} \cup \{ (a, b) | a \in A, b \in A, a \neq b \}\), and
let \(G_a (a \in A)\) be the subset of \(G\) that includes airspace \(a\), i.e. \(G_a := \{ g \in G | a \in A_g \}\). To
to judge the cost performance of an alliance, we use Algorithm 1 to determine capacity decision
\(x_g^* = (x_a^*)_{a \in A_g}\) with network costs \(\hat{f}(x_g^*)\) for each \(g \in G\). We can then determine the set
partition \(G\) for the cross-border setting with the following integer program:

\[
\begin{align*}
\min_{\theta} & \sum_{g \in G} \theta_g \hat{f}(x_g^*) \\
\text{s.t.} & \sum_{g \in G_a} \theta_g = 1 & a \in A \\
& \theta_g \in \{0, 1\} & g \in G
\end{align*}
\]

Decision variable \(\theta_g\) determines whether alliance \(g\) is used in the final set partition \(G\); we
have \(G = \{ g \in G | \theta_g = 1 \}\). The objective function (16) minimizes the total expected network
costs, and constraint (17) ensures that each airspace is contained in the final set partition.
To determine the alliances for the common tech setting we again solve the integer program
(16)–(18), but we restrict \(G\) to those alliances in which both airspaces use the same provider.
The set partitions used for all three settings are summarized in Table 9 in the Appendix.
Apart from the alliances, a further consideration in designing capacity sharing is the marginal cost for such sharing services. We require that the cost per sector-hour of providing capacity sharing is strictly higher than the local cost for ATM services, because ATCOs will need to be reimbursed for the additional qualification and training associated with this task. We set \( c^a := (1 + \rho)c_a \) for all \( a \in A \) and require for the cost markup \( \rho > 0 \). In particular, we set \( \rho := 0.1 \) in the baseline and compare different markups in a sensitivity analysis.

As discussed in §5.2.1, we assess the performance of each of the proposed capacity sharing settings with a two-staged simulation study: We first determine budget \( x^* \) using Algorithm 1 (using 300 training scenarios in \( S_1 \)), and test its performance using the XCILP* and Algorithm 2 (on a new set of 200 testing scenarios in \( S_2 \)). Again, simulations are run using AWS Batch with 4 CPU and 30GB RAM.

### 5.3.2 Discussion of results

Table 3 compares the cost performance in the simulation of the standard setting (without capacity sharing) with the three capacity sharing options, based on the network described in §5.1. We find that capacity sharing may reduce total network costs by €4,900 to €20,700 (around 2.0% to 8.4%) against the standard setting. Most notably, the total saving is realized with only small increases in capacity costs: In the cross-border setting, an investment of €430 (or 0.3%) in capacity costs leads to €8,248 (or 20%) lower displacement costs. The results also show that capacity sharing within the same ANSP (cross-ACC) delivers in fact larger savings than sharing among different ANSPs with the same technology provider (common tech). Since cross-ACC sharing would also be easier to implement, both politically and operationally, it should be the preferred setup. Note that one reason why we observe larger savings for cross-ACC sharing is that we allow alliances to contain up to four airspaces (since the German ANSP splits into four regions), while in common tech we allow only two.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Cap. cost</th>
<th>Displ. cost</th>
<th>Netw. cost</th>
<th>Savings</th>
<th>Sector-hours</th>
<th>Run time</th>
</tr>
</thead>
<tbody>
<tr>
<td>No sharing</td>
<td>140,936</td>
<td>106,112</td>
<td>247,048</td>
<td>-</td>
<td>912</td>
<td>126 min.</td>
</tr>
<tr>
<td>Cross-ACC</td>
<td>141,367</td>
<td>97,864</td>
<td>239,231</td>
<td>7,817 (3.2 %)</td>
<td>887 + 25</td>
<td>127 min.</td>
</tr>
<tr>
<td>Cross-border</td>
<td>141,413</td>
<td>84,911</td>
<td>226,324</td>
<td>20,723 (8.4 %)</td>
<td>875 + 37</td>
<td>173 min.</td>
</tr>
<tr>
<td>Common tech.</td>
<td>141,255</td>
<td>100,884</td>
<td>242,139</td>
<td>4,908 (2.0 %)</td>
<td>888 + 24</td>
<td>163 min.</td>
</tr>
</tbody>
</table>

The sector-hours reported in Table 3 show that out of the 912 total hours used in the standard setting, only a small fraction of 24 to 37 hours (or 2.6% to 4.1%) would need to be
deployed virtually in case of capacity sharing. This shows that the benefits from capacity sharing may be reaped with a relatively small amount of cross-border resources.

As expected, the solution times with capacity sharing are somewhat longer than for the standard setting, because the search space $\mathcal{X}$ becomes much larger. However, the run time stays below three hours in all settings, which allows practical implementation. Also note that the reported run times represent non-parallelized processes. By decomposing the problem, we can parallelize the solution process and solve it simultaneously for each airspace (or alliance, for capacity sharing). This would reduce the run time to around 8 minutes in the standard setting and 18-25 minutes for capacity sharing.

The savings in all settings are generated exclusively by reducing the average displacement costs rather than by reducing capacities. This is particularly relevant because lower re-routing costs also imply lower greenhouse gas emissions from reduced fuel burn. If an additional cost for emissions was considered, this would further increase the potential benefit reaped through capacity sharing. Figure 6 shows how the displacement costs vary for 50 of the 200 scenarios across the settings. We find that capacity sharing can reduce displacement costs especially when the costs in the standard setting are high. While costs under No sharing exceed €125,000 in 15 of the 50 scenarios (corresponding to cases with major disruption), the costs with cross-border sharing stay below this value for all but one scenario. This shows that the benefit of capacity sharing is especially large in case of major disruptions (e.g., due to adverse weather), in which case the displacement cost saving can amount to €30,000 or more. We also compare the performance to a lower bound on displacement costs, which we obtain by setting all capacities to their maximum historic reference values, i.e., $x_a^* := \bar{x}_a$ for all $a \in A$. We find that cross-border sharing realizes large parts of the total displacement cost reduction potential (on average over 68%) between the standard setting and the lower bound.

We also want to analyze which flights in particular benefit from the flexibility gained through capacity sharing. For this purpose, we compare the average displacement costs per flight for scheduled vs non-scheduled flights and different aircraft sizes between the standard setting and cross-border sharing (across five selected scenarios), see Table 4. The matching of aircraft types (e.g., Boeing 737) to small, medium or large aircraft size is shown in Table 8 in the Appendix. We find that flights with small aircraft benefit more from capacity sharing (-43.7%) than flights with medium- to large-sized aircraft (-14.1% to -15.1%). This is likely because without capacity sharing, small aircraft are re-routed or delayed more heavily (in the case of capacity shortages) because their displacements are less costly than those of medium or large-sized aircraft; with capacity sharing, these displacements often become unnecessary,
which disproportionately benefits small aircraft. Furthermore, we find that non-scheduled flights benefit more from capacity sharing (−28.4% in displacement costs) than scheduled flights (−14.1%). This is because non-scheduled flights are more likely than scheduled flights to operate on small aircraft, which benefit more from capacity sharing. Note also that the absolute displacement costs per scheduled flight is higher than for non-scheduled flights because scheduled flights are more likely to operate on large aircraft.

Table 4. Avg. displacement cost (EUR) per flight by setting (n = 5 scenarios).

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>Flight type</th>
<th>Setting</th>
<th>No sharing</th>
<th>Cross-border</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small</td>
<td>Medium</td>
<td>Large</td>
<td>Scheduled</td>
</tr>
<tr>
<td>Small</td>
<td>47.7</td>
<td>31.2</td>
<td>85.7</td>
<td>38.9</td>
</tr>
<tr>
<td>Medium</td>
<td>26.9</td>
<td>26.5</td>
<td>73.6</td>
<td>33.4</td>
</tr>
<tr>
<td>Large</td>
<td>50.0</td>
<td>38.9</td>
<td>73.6</td>
<td>33.4</td>
</tr>
<tr>
<td>Scheduled</td>
<td>50.0</td>
<td>38.9</td>
<td>73.6</td>
<td>33.4</td>
</tr>
<tr>
<td>Non-Scheduled</td>
<td>50.0</td>
<td>38.9</td>
<td>73.6</td>
<td>33.4</td>
</tr>
</tbody>
</table>

Finally, in Figure 7 we illustrate how capacity sharing helps increase the flexibility with which capacities can be adjusted within an alliance. The capacities used by the two Swiss airspaces (LSAZUTA and LSAGUTA) change frequently across the 50 scenarios, to adjust to the materialized demand and capacity provision.
Sensitivity Analysis

While capacity sharing can work to improve network performance, this comes at a cost. For that purpose, we want to determine what influence the marginal cost of capacity sharing has on the value of such a service. We conduct a sensitivity analysis with increased cost markups of $\rho := 0.2$ and $\rho := 0.4$ for the cross-border setting (which performed best in the tests above), see Table 5. Since the number of sector-hours required for capacity sharing is rather small (see Table 3), the total capacity costs increase only slightly if capacity sharing comes at a higher markup. In particular, total savings (compared to No sharing) decrease only from €20,724 to €20,247 (€19,293) for a markup of 20% (40%). This shows that if capacity sharing is implemented with only the few cross-border resources that we show are needed, then the marginal cost of such sharing services will not play a dominant role.

As in §5.2, we also test how well capacity sharing performs under different traffic inten-
Table 5. Sensitivity of cross-border capacity sharing to cost (n = 200 scenarios).

<table>
<thead>
<tr>
<th>Setting</th>
<th>Cap. cost</th>
<th>Displ. cost</th>
<th>Network cost</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>No sharing</td>
<td>140,936</td>
<td>106,112</td>
<td>247,048</td>
<td>-</td>
</tr>
<tr>
<td>Cross-border (10%)</td>
<td>141,413</td>
<td>84,911</td>
<td>226,324</td>
<td>20,724 (8.4%)</td>
</tr>
<tr>
<td>Cross-border (20%)</td>
<td>141,890</td>
<td>84,911</td>
<td>226,801</td>
<td>20,247 (8.2%)</td>
</tr>
<tr>
<td>Cross-border (40%)</td>
<td>142,844</td>
<td>84,911</td>
<td>227,755</td>
<td>19,293 (7.8%)</td>
</tr>
</tbody>
</table>

We find that cross-border sharing delivers the best results under low traffic, but cross-ACC sharing performs best under high traffic. Furthermore, the common tech setting shows comparatively low performance across all three traffic intensities. On our case study, this provides some evidence that establishing a shared technological infrastructure within alliances presents an important precondition to reap the benefits from capacity sharing. Note, however, that the savings from capacity sharing are not significant at 95% under either low or high traffic.

Table 6. Sensitivity of capacity sharing to traffic levels (n = 200 scenarios).

<table>
<thead>
<tr>
<th>Traffic level</th>
<th>Low Network cost</th>
<th>Savings</th>
<th>High Network cost</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>No sharing</td>
<td>166,369</td>
<td>-</td>
<td>386,800</td>
<td>-</td>
</tr>
<tr>
<td>Cross-ACC</td>
<td>163,144</td>
<td>3,225 (1.9%)</td>
<td>380,862</td>
<td>5,938 (1.5%)</td>
</tr>
<tr>
<td>Cross-border</td>
<td><strong>151,845</strong></td>
<td>14,525 (8.7%)</td>
<td>381,198</td>
<td>5,601 (1.4%)</td>
</tr>
<tr>
<td>Common tech.</td>
<td>162,722</td>
<td>3,647 (2.2%)</td>
<td>383,401</td>
<td>3,398 (0.9%)</td>
</tr>
</tbody>
</table>

Finally, Figure 8 compares the displacement cost performance of the standard vs cross-border setting for all three traffic levels and across 50 of the 200 evaluated scenarios. It is easy to see that the maximum potential to reduce displacement costs is lowest under low traffic and highest under high traffic (as reflected in the gap between No sharing and the lower bound). However, while cross-border sharing reduces costs to the lower bound in most of the scenarios under low traffic, it fails to do so under high traffic. This is likely because if traffic volumes exceed a certain level, all ATCOs will be required to manage local traffic and thus will not be available for capacity sharing. The results provide guidance under which conditions capacity sharing adds value: the expected traffic level needs to be large enough so that substantial displacements occur, but small enough so that airspaces do not require all of their ATCOs themselves.
Figure 8: Displacement cost saving of capacity sharing across traffic levels (n = 50 scenarios).

5.4 Limitations

The proposed methodology features some limitations. Firstly, while Algorithm 1 is scalable in the number of airspaces in the network, it does not scale well in the number of airspaces contained in each alliance (if capacity sharing is allowed). This is because we cannot decompose XCILP* by airspace but only by alliance. Secondly, to determine the regression parameters required for Algorithm 1, we need to run Algorithm 4 to generate a sufficiently large number of displacement cost observations. This procedure works well for the case study at hand, but does not scale well for much larger networks. Thirdly, we base our regression on estimates of displacement costs obtained from a heuristic procedure, which can only approximate these costs; this limits the quality of cost estimates obtained through regression (15).

Furthermore, there are a few assumptions made in the evaluation that may limit the generalizability of results. On the one hand, we assume that more sector-hours will lead to higher total ATCO costs in the long-term. However, this effect may only materialize inflexibly, since ATCOs are hired and trained years before becoming operational. On the other hand, we use a time horizon of 6 hours on one day to make strategic decisions (with regards to capacity levels and capacity sharing concepts), whereas in practice these decisions will need to be judged based on the performance over a sustained period of time. Finally, further practical considerations such as rostering practices for ATCOs are neglected in our simulation, but may have an impact on how well the strategic decisions translate into better network cost performance.
6 Conclusions

In European ATM, we observe large demand-capacity imbalances due to static capacities paired with large uncertainties in demand and capacity provision. To reduce the impact of these imbalances, we propose a capacity sharing scheme in which capacities can be shared among airspaces in an alliance. We develop an efficient approach to determine capacity budgets for each airspace that perform well across various scenarios (of demand and capacity uncertainty). A simulation study on a realistically-sized network with around 3,000 flights shows that the stochastically-determined capacity decision significantly outperforms a deterministic benchmark. As additional benefit, stochastically-determined capacities can lead to a more stable network performance by reducing variation in displacement costs. We use the proposed methodology to test different settings for capacity sharing, and find in the case study that cross-border sharing can reduce network costs by up to 8.4%. The reduction potential does however depend on the design of the alliances and the traffic intensity. If alliances can only be formed among ANSPs that use the same technological infrastructure, the cost savings reduce to 2–3% in our case study. Furthermore, if traffic volumes increase beyond a certain threshold, the benefits from capacity sharing decline because each airspace requires their local ATCOs themselves.

Author Statement


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References


European Commission. Legal, economic, and regulatory aspects of atm data services provision and capacity on demand as part of the future European air space architecture, Dec. 2020.


A Appendices

A.1 Latin Hypercube Sampling

This section describes the Latin Hypercube Sampling procedure used to build random capacity samples, see Algorithm 3. Each iteration of this algorithm generates $\bar{\eta}$ different samples, stored into a $\bar{\eta} \times |A|$ matrix, $X$. Let us also define $X^a = \{\bar{x}^{\text{min}}_a, \bar{x}^{\text{min}}_a + \epsilon, \bar{x}^{\text{min}}_a + 2\epsilon, ..., \bar{x}^{\text{max}}_a\}$, with $\epsilon = (\bar{x}^{\text{max}}_a - \bar{x}^{\text{min}}_a)/\bar{\eta}$. Values $\bar{x}^{\text{min}}_a$ and $\bar{x}^{\text{max}}_a$ represents the minimum and maximum number of sector hours for airspace $a$. These values are defined by the structure of the airspace’s configurations and the length of the time horizon.

Algorithm 3 Latin Hypercube Sampling

```
for $\eta = 1$ to $\bar{\eta}$ do
    $A \leftarrow A$
    while $A \neq \emptyset$ do
        $a$ is uniformly drawn and removed from $A$
        $\bar{x}_a$ is uniformly drawn and removed from $X^a$
        $X[\eta][a] = \bar{x}_a$
    end while
end for
return $X$
```

A.2 Heuristic for routing problem

Algorithm 4 MMKP-based heuristic for routing problem

```
Input: Configuration $C'$, traffic scenario $F^S$ and capacity uncertainty $W^S$
1: Initialize: Set $r'_f := \arg\min_{r \in R_f} d^L_f$ for $f \in F^S$, Lagrange Multiplier $\mu_l := 0$ for $l \in L'$
2: Establish feasible solution: Iterate until $\bar{k}_l \leq 1\forall l \in L'$
3: Compute relative “weight” $w_{fr} = \sum_{e \in E_l} b_{free}/\kappa_i^S$ for $f \in F^S, r \in R^f, l \in L'$
4: Compute relative capacity shortage $k_l = \sum_{f \in F^S} w_{fr} - \sum_{l \in L'} \mu_l (w_{fr} - w_{fr}^*)$ for $r \in R^f$
5: For flights with $w_{fr} > w_{fr}^*$ on $l^*$, store $\gamma^l_f = d^L_f - d^L_f - \sum_{l \in L'} \mu_l (w_{fr} - w_{fr}^*)$ for $r \in R^f$
6: Determine flight and route with lowest $\gamma^l_f$, update $r'_f = r$ and $\mu_{alu^*} = \mu_{alu^*} + \gamma^l_f$
7: Improve feasible solution: Iterate until no further improvement found, i.e., $\Delta d = \emptyset$
8: For flights and routes with $d^L_f > d^L_f$ and $\bar{k}_l - w_{fr} + w_{fr}^* \leq 1, l \in L'$, store $\Delta r = d^L_f - d^L_f$. 
9: Find flight and route with largest $\Delta r$ and update $r'_f := r$
Output: Routing $R^* = \{r'_f : f \in F^S\}$ and displacement cost $D^* = \sum_{r \in R^*} d_r$
```
### A.3 Case study data

Table 7. Capacity-side network characteristics in the case study

<table>
<thead>
<tr>
<th>Airspace</th>
<th>Elementary sectors</th>
<th>Collapsed sectors</th>
<th>Configurations</th>
<th>Probability of ATFM regulation</th>
<th>Technology provider</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDUUUTAC</td>
<td>11</td>
<td>14</td>
<td>13</td>
<td>10.8%</td>
<td>Indra</td>
</tr>
<tr>
<td>EDUUUTAE</td>
<td>10</td>
<td>14</td>
<td>13</td>
<td>8.3%</td>
<td>Indra</td>
</tr>
<tr>
<td>EDUUUTAS</td>
<td>12</td>
<td>29</td>
<td>13</td>
<td>41.6%</td>
<td>Indra</td>
</tr>
<tr>
<td>EDUUUTAW</td>
<td>10</td>
<td>12</td>
<td>11</td>
<td>2.5%</td>
<td>Indra</td>
</tr>
<tr>
<td>EDYYBUTA</td>
<td>8</td>
<td>13</td>
<td>10</td>
<td>0.0%</td>
<td>Indra</td>
</tr>
<tr>
<td>EDYYDUTA</td>
<td>9</td>
<td>12</td>
<td>7</td>
<td>0.0%</td>
<td>Indra</td>
</tr>
<tr>
<td>EDYYHUTA</td>
<td>12</td>
<td>19</td>
<td>12</td>
<td>0.0%</td>
<td>Indra</td>
</tr>
<tr>
<td>EPWWCTA</td>
<td>18</td>
<td>77</td>
<td>26</td>
<td>56.8%</td>
<td>Indra</td>
</tr>
<tr>
<td>LHCCCTA</td>
<td>10</td>
<td>24</td>
<td>7</td>
<td>0.0%</td>
<td>Thales</td>
</tr>
<tr>
<td>LKAACCTA</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>0.2%</td>
<td>Legacy</td>
</tr>
<tr>
<td>LKAAMUTA</td>
<td>6</td>
<td>9</td>
<td>8</td>
<td>0.0%</td>
<td>Legacy</td>
</tr>
<tr>
<td>LOVVCTA</td>
<td>26</td>
<td>58</td>
<td>21</td>
<td>1.2%</td>
<td>Thales</td>
</tr>
<tr>
<td>LSAGUTA</td>
<td>6</td>
<td>9</td>
<td>11</td>
<td>3.1%</td>
<td>Legacy</td>
</tr>
<tr>
<td>LSZUTA</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>0.3%</td>
<td>Legacy</td>
</tr>
<tr>
<td>LZBBCTA</td>
<td>27</td>
<td>69</td>
<td>8</td>
<td>0.0%</td>
<td>Thales</td>
</tr>
</tbody>
</table>

Technology providers taken from Eurocontrol (2022); ”legacy” implies that ANSP employs its own system.
Table 8. Aircraft size matching based on ICAO classification.

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>Aircraft size</th>
<th>ICAO Wake Turbulence Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>B737</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>B738</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>B75</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>B76</td>
<td>large</td>
<td>H (heavy)</td>
</tr>
<tr>
<td>B74</td>
<td>large</td>
<td>H (heavy)</td>
</tr>
<tr>
<td>A319</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>A320</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>A321</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>AT4</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>AT7</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>DH8</td>
<td>medium</td>
<td>M (medium)</td>
</tr>
<tr>
<td>E179</td>
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<td>M (medium)</td>
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<td>A33</td>
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<td>A34</td>
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<td>B78</td>
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</tr>
<tr>
<td>BE20</td>
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</tr>
<tr>
<td>C560</td>
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<tr>
<td>SMP</td>
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<td>L (light)</td>
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<tr>
<td>EXMT</td>
<td>small</td>
<td>L (light)</td>
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SMP refers to other small aircraft types, and EXMT refers to flights typically exempt from demand management measures.
Table 9. Geographic setup for capacity sharing settings.

### Cross-ACC

<table>
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<tr>
<th>Alliance</th>
<th>Airspaces</th>
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<tr>
<td>8</td>
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<td>9</td>
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### Cross-border

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### Common technology

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<td>LOVvCTA (Austria)</td>
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</table>

Germany abbreviated by “G.”.